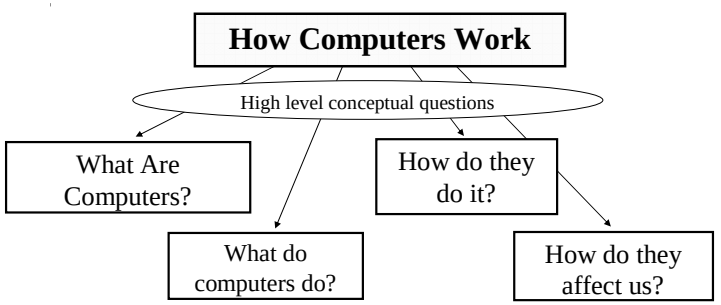
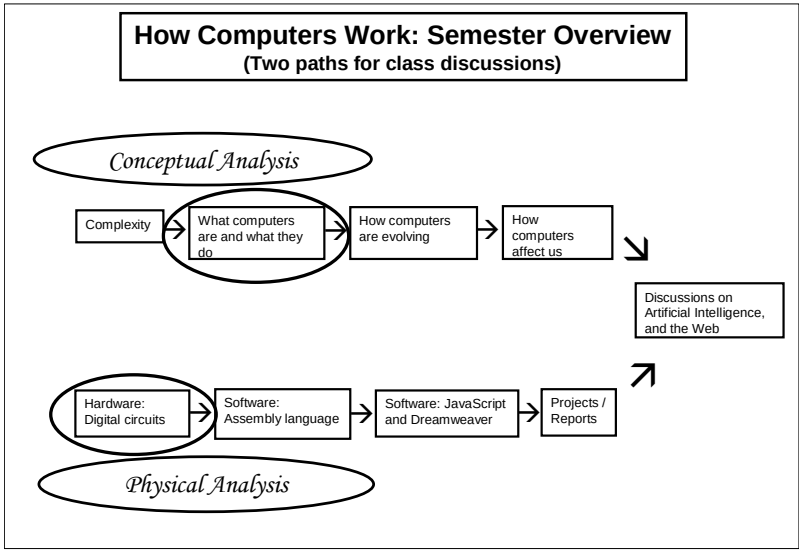
Binary Addition

Boolean Algebra & Logic Gates

CSC 103

September 12, 2007





Recap from Monday

- Binary numbers (§ 1.1.1)
  - The computer stores everything as binary numbers
    - Data – numbers, text, music...
    - Instructions for programming
    - Locations / addresses

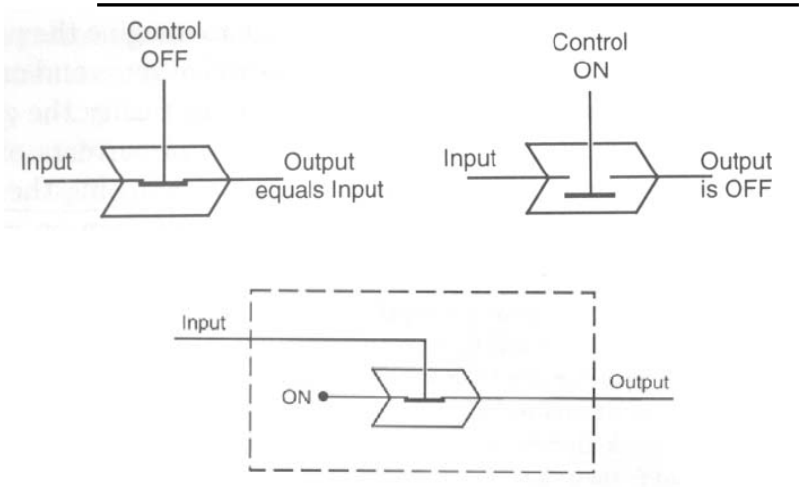
Overview for Today

- Binary concepts and binary numbers
- Circuit simulator
- Binary addition
  - Truth table
  - Half adder circuit

Binary Numbers: Physical Representation

- Concept of “on” and “off” for physical manufacturing of computers
- Basic component
  - Switch ⇨ Allows control of “on” & “off”
  - Historical versions?
    -
    -
  - Current implementation – the transistor

Transistors



Binary Numbers: Abstract Representation

- Logic
  - Provides the primary logic functions of AND, OR, NOT
- Logic gates & logic circuits
  - Symbolic representation of basic logic functions to build circuits

Logic Functions: NOT

- One input and one output
  - Output = input *or*
  - Output ≠ input
- The 'NOT' function

A	A'
0	1
1	0

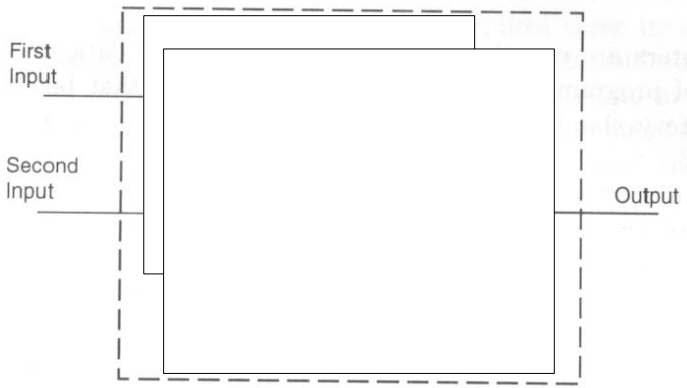
Logic Functions: AND, OR

- Two inputs, one output
- The 'AND' and 'OR' functions

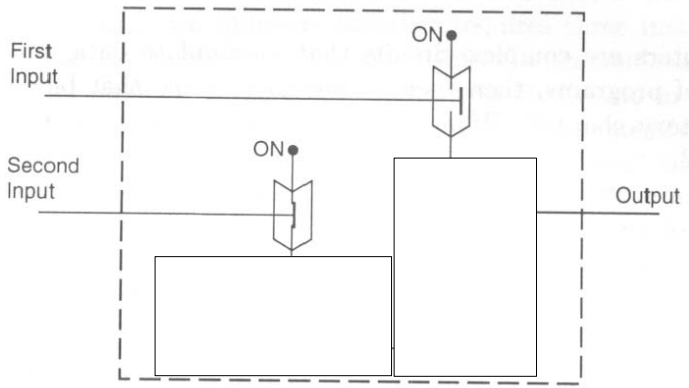
A	B	AND (·)	OR (+)
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

- This is a *truth table*

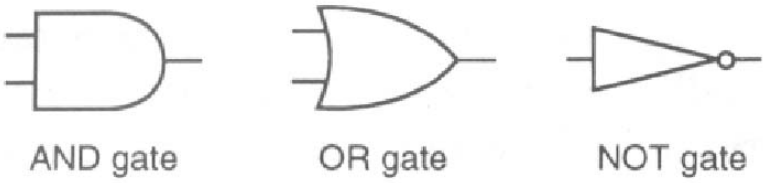
Logic Gate: AND Function



Logic Gate: OR Function



Logic Gates: Symbols



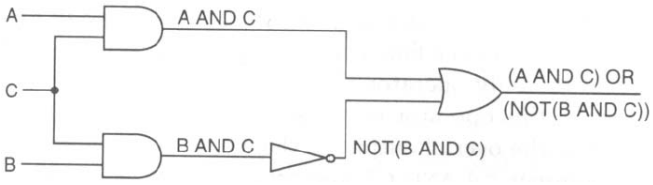
AB, $A \cdot B$	$A+B$	$\overline{A}$, A'

Use of Logic Gates

- Data is represented by 1s and 0s
  - '1' means there is current or voltage
  - '0' means there is no voltage or current
- Digital circuits (*e.g.*, MP3 players, cell phones, computers)
  - These circuits are ***all*** constructed with AND, OR and NOT gates
- ***ANY*** digital circuit can be built with AND, OR and NOT gates

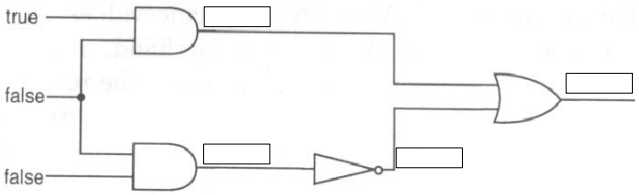
Logic Circuit Example

- How do we implement the following function as a circuit?
  - AC + (BC)'
  - In words this is:
  - As a circuit this is:

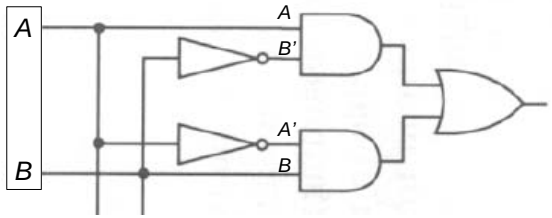


Logic Circuit Example con't

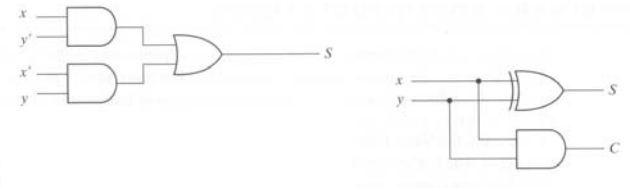
- Implementing the following function
  - AC + (BC)'
  - A = 1 *or* true
  - B = 0 *or* false
  - C = 0 *or* false



Exclusive OR



Exclusive OR

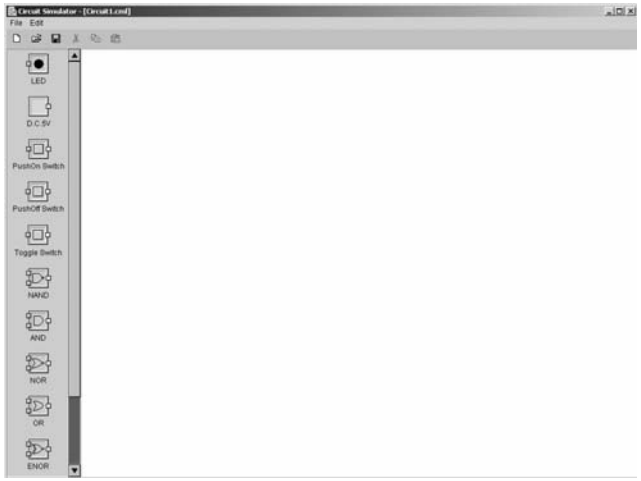


The Circuit Simulator

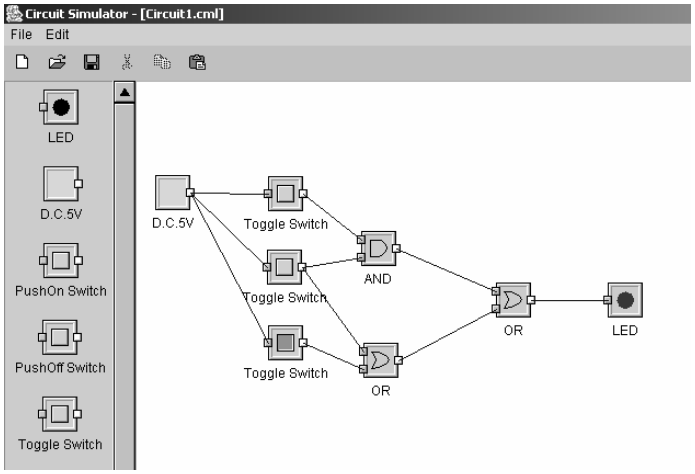
Circuit Simulator

- Go to Exercise 1 on the course webpage
  - Do Exercise 1 of this page (notice that the rest of the page is the homework)
- To get started, download the Circuit Simulator (SimCir) zip file
- Select "Open Using Winzip"
- When the zip file opens, double click simcir.jar
- Circuit Simulator link

Circuit Simulator Window



Circuit Example



Using the Simulator

- Include a power supply (only need one)
- Use toggle switch for inputs
- Use LED for output (red=1, black=0)
- Quirks
  - If you resize, it will jumble your circuit
  - To cut – select, and then click on the scissors
  - Some people have trouble saving – test this first!!

Onto the Adder Circuit...

Addition

- Adding decimal numbers
  - Concept of the carry digit

```
  15
  27
  42
```

- Adding binary numbers
  - Concept of sum and carry bits

Addition Concepts

- Two inputs – two numbers to add, digit by digit (bit by bit)
  - *AND*, there is a carry, which we represent as a third input
- Two outputs
  - Sum
  - Carry
- 'Carry' is an output from one step, and an input to the next one

Binary Addition

- Add
  - 0 + 0 =
  - 0 + 1 =
  - 1 + 0 =
  - 1 + 1 =
- Add these numbers

c:

1000111	1011010	1011010
0100110	1000111	0111001

s:

Binary Addition: Half Adder

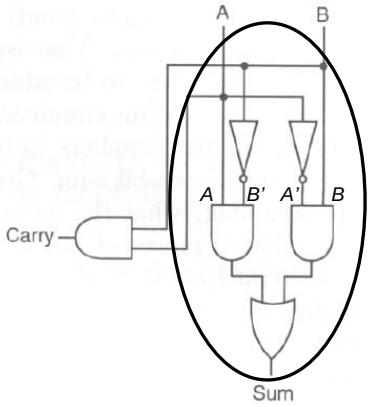
- We need a circuit to add two bits
  - Either bit can be '0' or '1'
- The function in the truth table is
  - Sum = A'B + AB'
  - Carry = AB

A	*B*	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Binary Addition: Half Adder

- A circuit with two inputs and two outputs

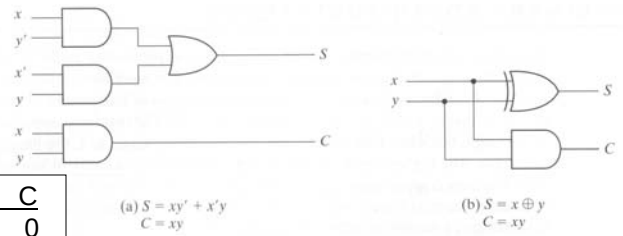
A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



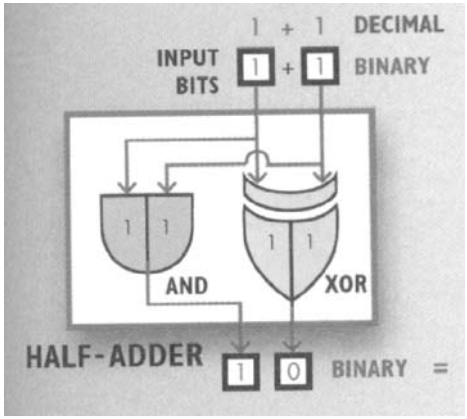
The Half-Adder and Exclusive OR Gate

- A'B + AB' = Exclusive OR
  - Typically abbreviated to XOR
  - Simulator uses EOR

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



Binary Addition: Half Adder



Binary Addition: Full Adder

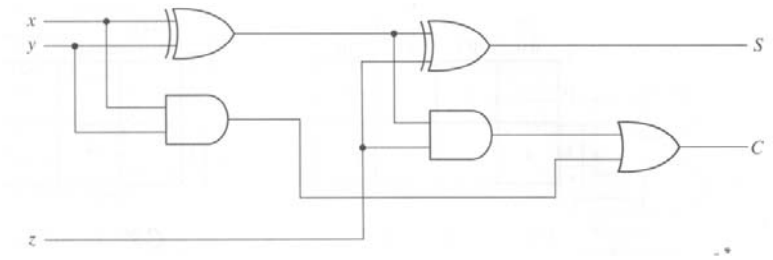
- A full adder is a circuit with *three* inputs (including a 'carry-in') and two outputs (the sum and carry-out)
  - What is the third input?
  - Exercise: Add 111+ 101

(carry)
1 1 1 ('A')
1 0 1 ('B')
(sum)

- For adding two numbers, we need three inputs

Binary Addition: Full Adder

- Cascade two half-adders to get a full adder



Summary

- Digital logic circuits
  - Transistors/Switches
  - Binary numbers
  - Boolean algebra
  - Propositional logic
- Primary logic functions
  - AND, OR, NOT
- Binary addition
  - Half adder circuit

Administration

- Class slides will be posted
- Quiz 1 – posted, and is due at midnight Sunday.
- Office hours and TA